



# Development of a Computer Program for Geometric Characterization of Heat pipe based Compound Parabolic Concentrator

\*Abdullahi, B., Muaz, N.M. and Tambaya, M.

Mechanical Engineering Department, Kano University of Science and Technology, Wudil, Kano- Nigeria

**Abstract** - Two models were developed for generating, characterizing and detailed analysis of different geometries of heat pipe based compound parabolic collector (HPCPC) with acceptance angle between  $20^\circ$  and  $60^\circ$ . A computer program was developed in Microsoft Excel (called HPCPCGeometric) which generates the concentrator profile and characterizes it in terms of height, aperture width, height to aperture ratio, truncation effects, concentration ratio and average number of reflections using different combinations of acceptance angles and receiver radii. Results have shown that the collector height, aperture width and concentration ratio increase as acceptance angle decreases. Hence it shows that the acceptance angle of the HPCPC is the most important parameter in determining its geometric characteristics because it affects the collector height and aperture, concentration ratio, average number of reflections, etc. The same correlations were coded in Solidworks® and the HPCPCs generated gave the same properties as those obtained from the HPCPCGeometric model. New correlations relating collector height, aperture width and acceptance angles were generated using the data obtained from such models.

**Keywords:** Compound Parabolic Collector, acceptance angle, receiver radius, concentration ratio, Concentrator

## 1.0 INTRODUCTION

Solar thermal collectors are heat exchangers that transform solar radiation into heat and transport the heat to a working fluid (water, air, oil, etc.) flowing through it. They consist of different components such as glazing, receiver, receiver plate and concentrator (in case of concentrating systems). There are different types of collectors which are categorized based on their motion (i.e. can be stationary, one axis or two axes tracking collectors), design (concentrating or non-concentrating) or operating temperatures. Also concentrating collectors can be divided into imaging and non-imaging. When the sun's image is formed on the receiver, it is called imaging collector which includes parabolic dish, parabolic trough, central receiver (or heliostat field collector) and linear Fresnel reflector (LFR) while the compound parabolic concentrator belongs to the non-imaging category. Non-imaging concentrators can meet low and medium concentration without tracking the sun and this makes them more affordable than imaging counterpart. Compound Parabolic Collectors (CPC) are non-imaging collectors which have high concentration ratio for a given acceptance angle. It concentrates all solar radiations falling on it within a wide range of angle (acceptance angle). CPCs have properties of both concentrating and flat plate collectors with low errors of alignment for reflective and receiving surfaces and high optical efficiency.

Figure 1 shows a 2 D view of the compound parabolic collector (CPC) which consists of the concentrator, receiver and glazing. The performance of solar collectors is greatly influenced by their geometries such as height, aperture width, concentration ratio, receiver sizes, etc. After the first design of CPC by Winston in 1974 (Winston, 1974), research works were conducted using different design approaches to investigate its capabilities and advantages for different applications. Details analysis on the general characteristics of the CPC in terms of concentration, acceptance angle, average number of reflection, sensitivity to mirror error, operating temperature, etc. was presented by Rabl, (1975) and Rabl et al., (1979). A 3-D CPC with acceptance angle of  $4^\circ$  was fabricated using horizontal segment technique to overcome the short comings of 2-D profile (Senthilkumar and Yasodha, 2012).

The geometric and optical properties of fully illuminated inverted V shape receiver CPC with vertex angle greater or equal to the acceptance half angle, was analytically studied by Fraidenraich et al., (2008). Calculations of the energy obtained and cost benefit analysis were also carried out for different collector orientations and various vertex and acceptance angles. According to the authors, the results obtained has led to the best collector design for the Recife – PE city, Brazil with the concentration ratio between 1.0 and 1.2 for East – West orientation, acceptance and absorber vertex angles of  $30^\circ$  and  $65^\circ$  respectively. Prototypes of six different collectors for stand-alone, roof and wall

\*Corresponding author Tel: +234-8033339805  
Email: [ibniabdullah12@gmail.com](mailto:ibniabdullah12@gmail.com)



mounting were built and tested outdoor by Adsten et al., (2005).

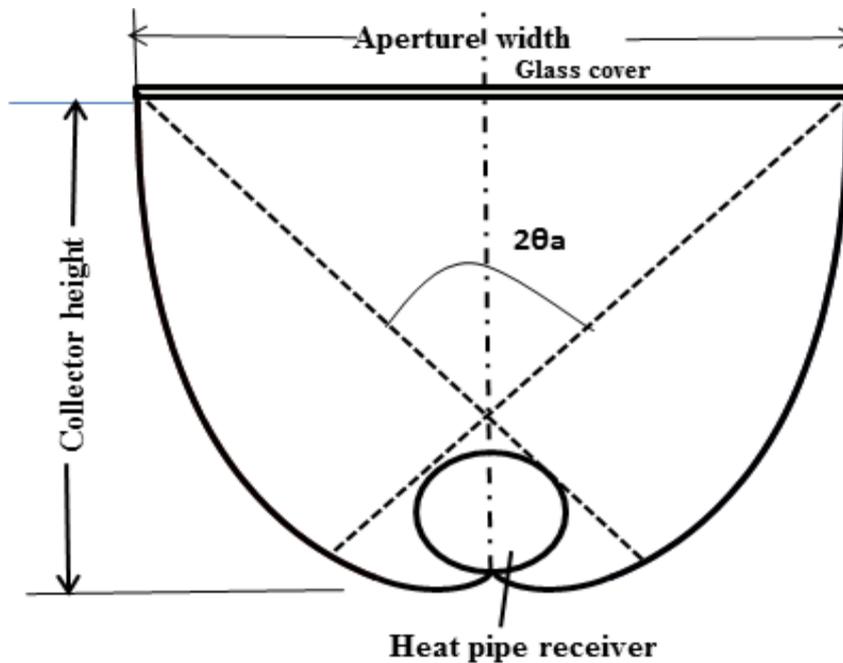


Figure 1: Compound Parabolic Collector

Results showed that the roof mounted collector gave the highest annual energy output of  $925 \text{ MJ/m}^2$  and the stand-alone with Teflon produced  $781 \text{ MJ/m}^2$ . Two different types of solar collectors; a standard glazed flat plate and evacuated tube collector with external compound parabolic concentrator reflector were compared by measuring their efficiencies in steady-state and partial dynamic conditions using EN12975-2 test standards by Zambolin et al., (2010). The work included experimental measurements of the two collectors in terms of generation of efficiency curves, daily efficiencies and characterizing the collectors based on the daily efficiencies obtained. Theoretical and experimental results showed that the evacuated tube with CPC reflector produced higher daily efficiency than the flat plate collector. But it is noted that the two collectors compared have different gross and aperture areas of ( $5.16 \text{ m}^2$  and  $3.9 \text{ m}^2$ ) and ( $4.76\text{m}^2$  and  $3.5\text{m}^2$ ) for flat plate and evacuated tube respectively. A low concentration CPCs with geometric concentration ratio of 2.92 and 1.95 for full and truncated reflector respectively were presented by Nkwetta et al., (2012). The receiver is tubular evacuated heat pipe of 15mm diameter enclosed in a white borosilicate glass tube. A 2D ray tracing technique was used in analysing the performance of such collector in terms of flux distribution, optical losses and efficiency at different incident angles. Results predicted 93.72% and 79.13% ray's acceptance and optical efficiency respectively between transverse angle of  $0^\circ$  and  $20^\circ$ . Since this work considered very small range of incident angles, it could

have been good to explain if the intended collector will be adjusted periodically. Also since the authors used in house software, there is need for validation either by experiment or by using more established software. The thermal performance of an evacuated heat pipe tube integrated into CPC was tested outdoor according to the ISO 9806 – 1 standard by Chamsa-ard et al., (2014). A numerical model was developed to predict the energy production based on the atmospheric condition of Phitsanulok, Thailand. Their results showed 78%, 3.55 and  $0.06 \text{ W/(m}^2\text{C)}$  thermal efficiency and heat loss coefficients  $a_1$  and  $a_2$  respectively and 286.16 kWh monthly energy production.

Although several attempts were made in studying the characteristics of different collectors, but computer programs for fully characterizing heat pipe based CPC is still limited. Also it should be noted that CPC with heat pipe as its receiver has advantage over those using other pipes in terms of thermal efficiency due to the high heat transfer characteristics of heat pipe. So developing a computer program for analysing its performance when coupled with heat pipe receiver is of paramount important. Also the study will add knowledge on the performance enhancement of CPC for economic and other reasons. Hence this paper is aimed at developing a model for geometric characterization and generating its profile for different inputs.

## 2.0 CPC PROFILE

The acceptance angle ( $2\theta_a$ ) and the receiver radius ( $r$ ) of CPC are very important parameters in defining its



geometric and optical properties. These two parameters are used in generating the geometry of CPC and they define other factors related to the collector such as the concentration ratio, average number of reflection of the rays and the geometry size (collector height (H), aperture width ( $A_w$ ) and height to aperture ratio). The acceptance angle of CPC is the maximum angle within which all radiations are concentrated on the receiver. Hence the gain and loss of rays by the CPC depends on the incident angle of the rays compared with the acceptance half-angle ( $\theta_a$ ) of the CPC.

The geometry of symmetric CPC can be generated by using the following equations (Chaves, 2008):

For involute section;

$$\begin{aligned} x &= r(\pm \varphi \cos \varphi \pm \sin \varphi) \\ y &= r(-\cos \varphi - \varphi \sin \varphi) \end{aligned} \quad (1)$$

$$\text{Where } -\left(\frac{\pi}{2} + \theta_a\right) \subseteq \varphi \subseteq 0$$

And for macrofocal parabola section;

$$\begin{aligned} x &= \frac{r}{\cos \varphi - 1} \left[ \pm \cos \theta_a \pm \cos(\varphi - \theta_a) \pm \right. \\ &\quad \left. (2\pi - \varphi + 2\theta_a) \sin(\varphi - \theta_a) \right] \\ y &= \frac{r}{\cos \varphi - 1} \left[ (-2\pi + \varphi - 2\theta_a) \cos(\varphi - \theta_a) \right. \\ &\quad \left. - \sin(\varphi - \theta_a) - \sin \theta_a \right] \end{aligned} \quad (2)$$

Where  $\varphi$  is the term which is defined by the range of the acceptance angle for the macrofocal and involute sections to join together, and it is given by  $2\theta_a \subseteq \varphi \subseteq \pi$

Where  $\theta_a$  and  $r$  are the half acceptance angle and receiver radius respectively. While  $x$  and  $y$  are the coordinates plotted to achieve the desired shape and size of the concentrator.

The measurement of the ability of CPC to concentrate solar radiation onto its receiver is termed as the geometric concentration ratio,  $C$  which is mathematically defined as the ratio of the collector aperture area to the area of the receiver, as (Rabl, 1985):

$$C = \frac{A_a}{A_r} \quad (3)$$

Also the geometric concentration ratio,  $C$  is related to the half acceptance angle as (Rabl, 1985):

$$C = \frac{1}{\sin^2 \theta_a} \quad (4)$$

The incident ray passing through the aperture of the CPC with angle less than the acceptance angle of CPC is reflected onto the receiver directly or after one or more reflection on the concentrator. The average number of reflections,  $n_i$  for full and truncated CPC is calculated as (Duffie and Beckham, 2006):

$$n_i = \max \left[ C \frac{A_{RT}}{4a_T} - \frac{x^2 - \cos^2 \theta}{2(1 + \sin \theta)}, 1 - \frac{1}{C} \right] \quad (5)$$

Where “max” is the maximum number of reflections obtained from the two correlations;  $C \frac{A_{RT}}{4a_T} - \frac{x^2 - \cos^2 \theta}{2(1 + \sin \theta)}$  and  $1 - \frac{1}{C}$  in equation 5 is considered.

And  $x$  is given as (Duffie and Beckham, 2006):

$$x = \left( \frac{1 + \sin \theta}{\cos \theta} \right) \left( -\sin \theta + \left( 1 + \frac{h_t}{h} \cot^2 \theta \right)^{1/2} \right) \quad (6)$$

Where  $A_{RT}$  is the reflector area per unit depth of the truncated CPC while  $h_t$  and  $h$  are truncated and full height of the concentrator respectively.

## 2.1 CPC Profile Model

A computer program was developed in Excel spreadsheet using equations 1 to 5 for generating and characterizing the HPCPC. The HPCPC formed from this model is made up of four segments; positive and negative involute as well as positive and negative macrofocal, joined together as shown in Figure 2. Each segment is generated by  $x$  and  $y$  coordinates based on equations 1 and 2, using the receiver radius and the range of the  $\varphi$  (which depends on the acceptance angle). Figure 3 shows a flow chart summarizing the modelling procedure for characterizing the collector.

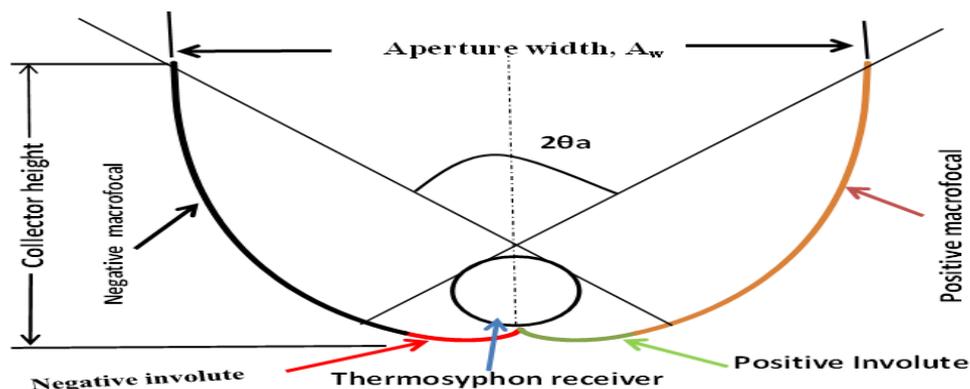
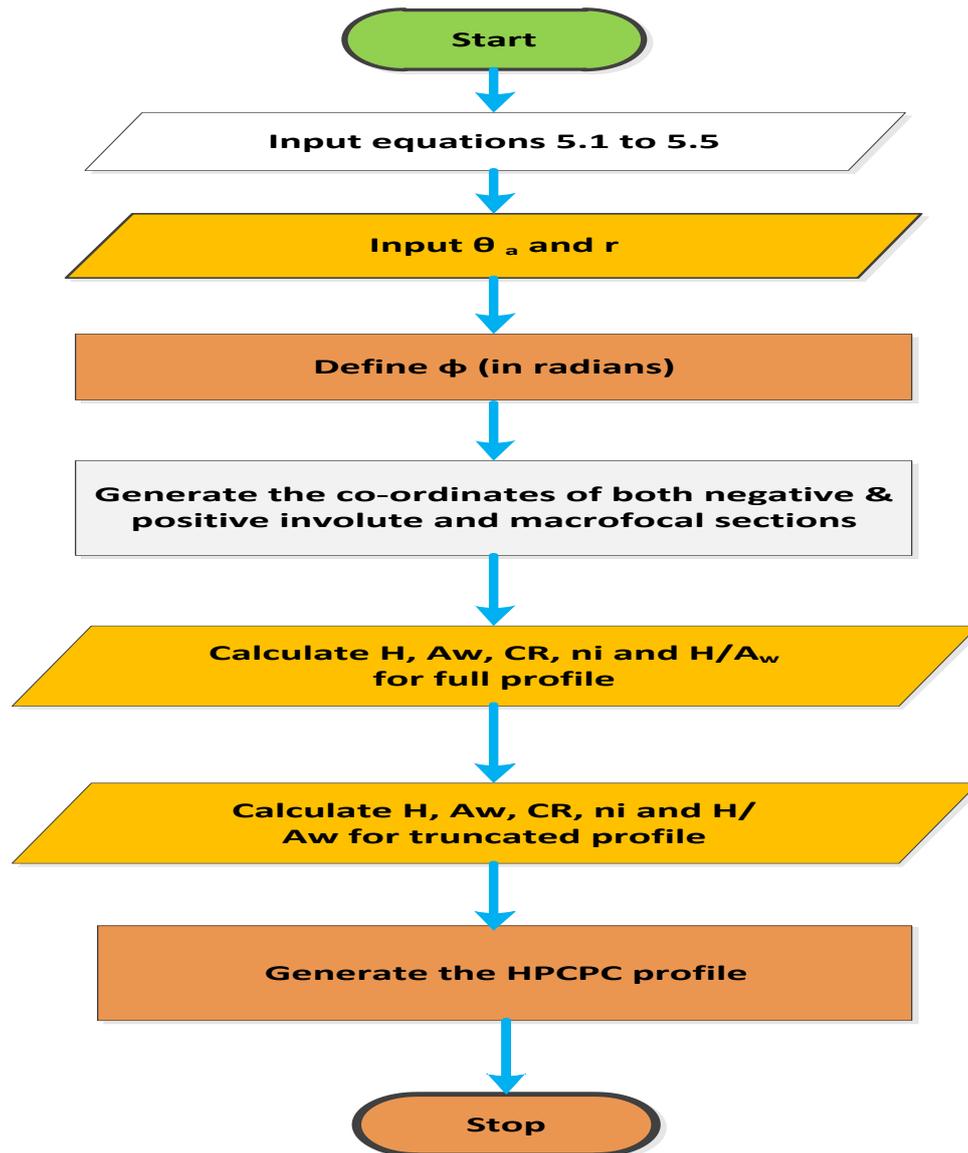


Figure 2: 2-D view of the HPCPC



**Figure 3:** Flow chart of the HPCPC geometric model

The generated coordinates are plotted using scatter chart type to obtain the geometry, and once the receiver radius is changed for a particular acceptance angle, the values in the coordinates and the generated HPCPC are updated to give a new shape and dimensions. Other outputs obtained are the concentration ratio, collector height, aperture width, height-to- aperture ratio and average number of reflections. In generating the HPCPC for this work, acceptance angle and radius of the receiver are considered as the inputs. Acceptance angles ( $2\theta_a$ ) ranging from  $20^\circ$  to  $60^\circ$  and receiver radii ( $r$ ) of 0.011m, 0.0125m, 0.0225m and 0.025m were used in this study. The selection of the acceptance angle is based on the target concentration ratios and the receiver radii are based on the commercially available pipe sizes and the intended applications.

### 3.0 RESULTS AND DISCUSSION

Using the model developed in 2.1 and the acceptance angles and receiver radii stated, the geometries of different HPCPCs were generated and characterized based on the collector height, aperture width, concentration ratio, height to aperture ratio, etc. Figure 4 shows two dimensional views of the HPCPCs generated from this model at different acceptance angles and 12.5mm receiver radius. Also to verify the geometric parameters of the generated HPCPC obtained from this model, the parametric equations 1 and 2 were coded in the Solidworks<sup>®</sup> software and twenty different geometries were generated and characterized. Figure 5 shows a 3D view of one of such geometry obtained from solid works. The results from the two approaches were found to be the same and the characteristics of some of these geometries are shown in Table 1.

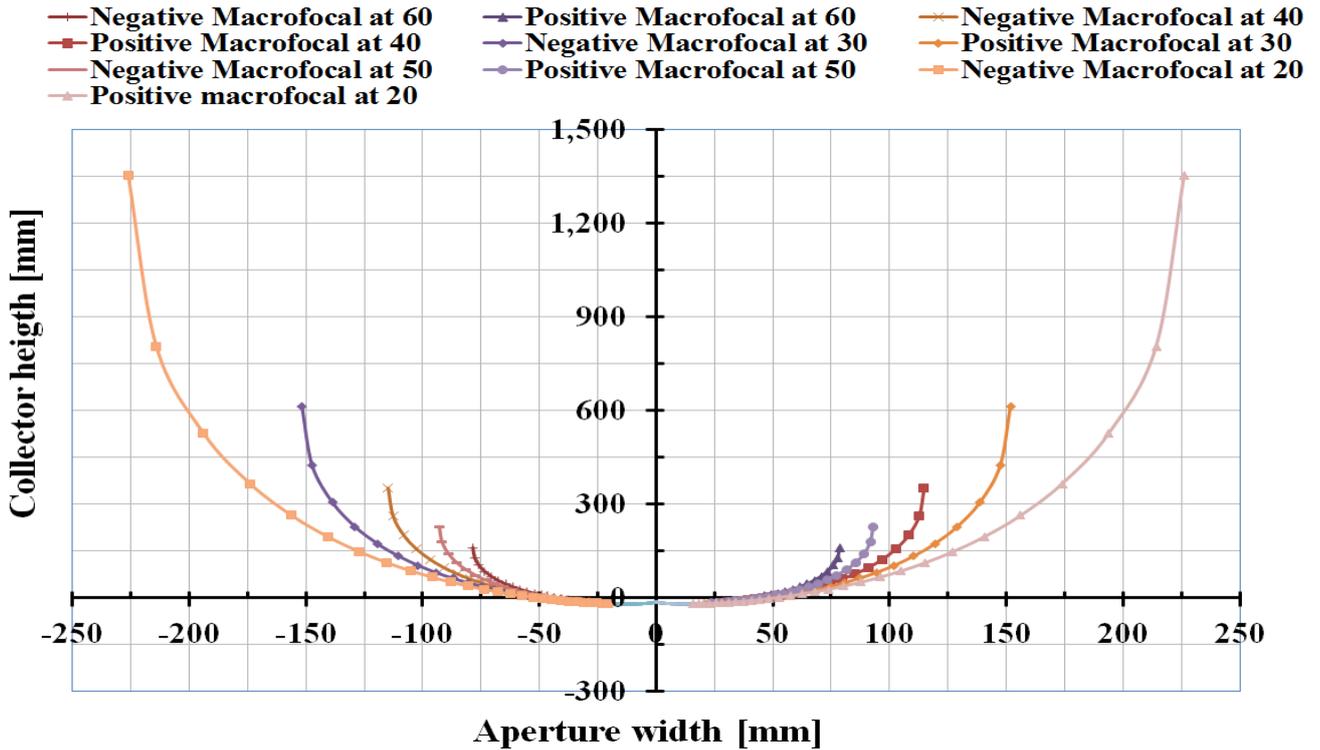


Figure 4: Two dimensional views of the generated CPCs at different acceptance angles and receiver radius of 12.5mm

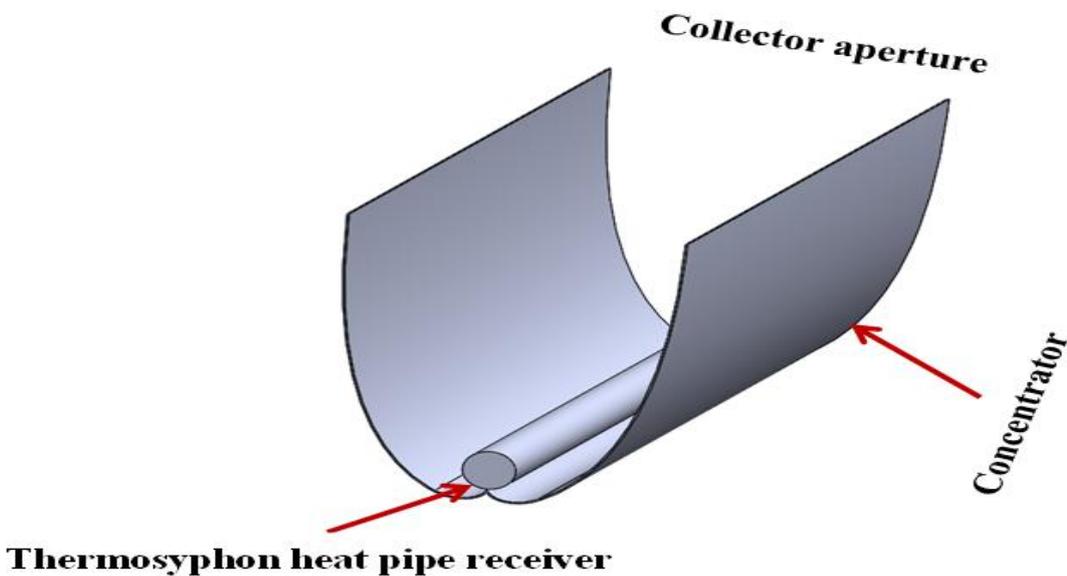


Figure 5: 3D view of the HPCPC generated from solidworks

Figure 6 shows the effects of the CPC acceptance angle and the receiver radius on the height of the collector. The height of the collector decreases as the acceptance angle increases when the receiver radius is kept constant; thus 60° acceptance has the shortest

concentrator for each receiver size. Figure 7 shows the effects of the acceptance angle and receiver radius on the aperture width of the HPCPC. It can be seen from such figure that the aperture width increases as the acceptance angle decreases for any receiver size.

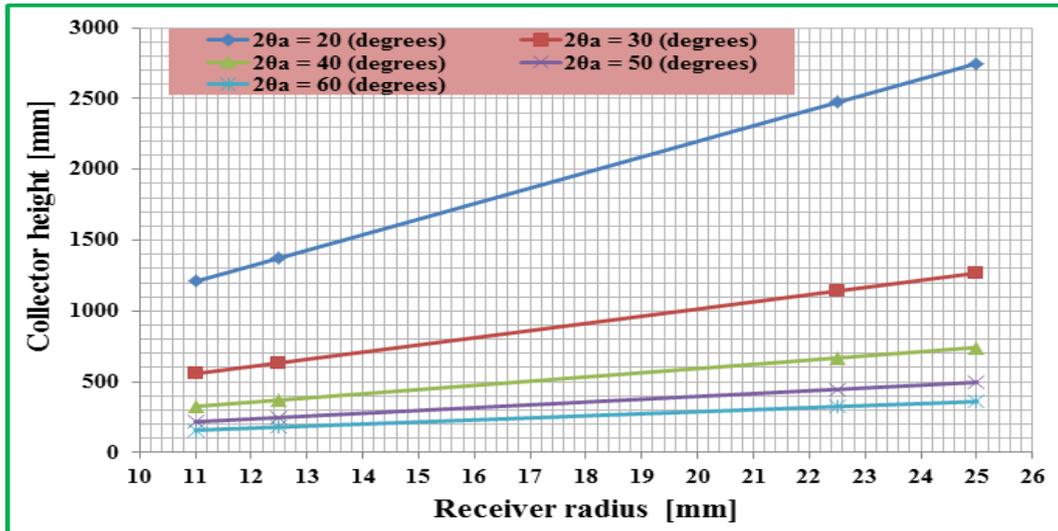


Figure 6: Variation of the CPC height with the receiver radius at different acceptance angles

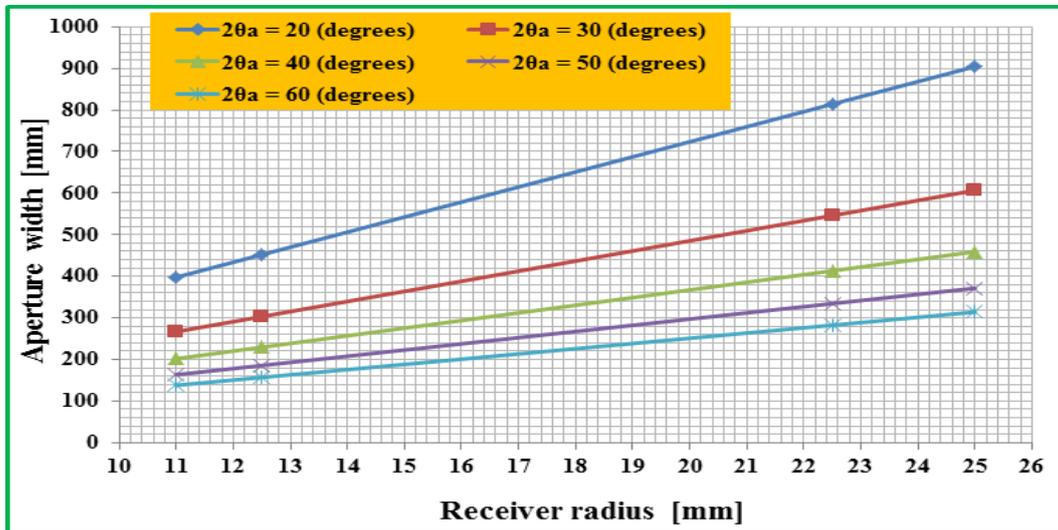


Figure 7: Variation of the CPC aperture width with receiver radius at different acceptance angles

Also, it can be deduced from Table 1, Figures 6 and 7 that both the collector height and the aperture width increase as the receiver radius increases. Furthermore, as the acceptance angle decreases both the collector height,

aperture width and the concentration ratio increase (Figure 8).

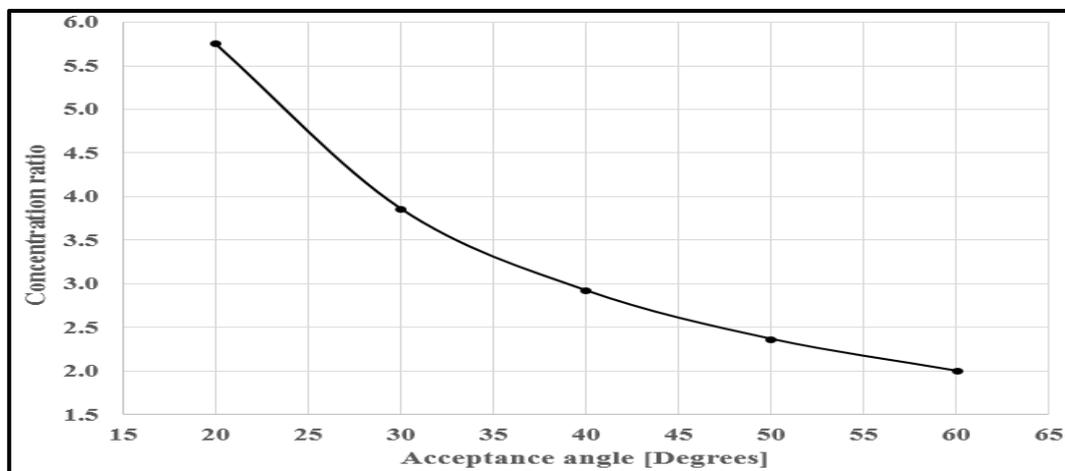


Figure 8: Variation of concentration ratio with the acceptance angle



**Table 1:** Characteristics of some of the CPCs generated from this model

Serial Number	Receiver radius (mm)	Geometric Parameters	Acceptance angle		
			60°	40°	30°
1	12.5	Collector height (mm)	180.66	371.617	634.171
		Aperture width (mm)	157.08	229.64	303.45
		Concentration ratio	2	2.92	3.86
		Height to aperture ratio	1.15	1.62	2.09
2	22.5	Collector height (mm)	325.796	668.91	1141.51
		Aperture width (mm)	282.74	413.34	546.22
		Concentration ratio	2	2.92	3.86
		Height to aperture ratio	1.15	1.62	2.09
3	25.0	Collector height (mm)	361.329	743.234	1268.34
		Aperture width (mm)	314.16	459.27	606.91
		Concentration ratio	2	2.92	3.86
		Height to aperture ratio	1.15	1.62	2.09

Hence a full CPC with small acceptance angle and large receiver tends to be bigger and have high concentration ratio compared to its counterpart of large acceptance angle and small receiver size. The results obtained from this model are in good agreement with those reported by Rabl, (1976) and Rabl et al, (1979) in their papers which describe the design, geometric and optical properties of CPC as well as the results published in his book Rabl, (1985).

Various combinations of acceptance angles (of 20°, 30°, 40°, 50° and 60°) and receiver radii (of 11mm, 12.5mm, 22.5mm, 25mm) were inputted into the model and various collector heights and aperture widths were obtained (from such combinations of the acceptance angles and the receiver radii) using the developed computer program. The collector heights obtained were plotted on X-Y scattered charts on excel spreadsheet

with ratio of the receiver radius to the height ( $r/h$ ) plotted against the acceptance angle where relation between the three parameters is obtained as equation 7. In the same way the relationship between the collector aperture width, acceptance angle and the receiver radius is also obtained as equation 8. These equations obtained are for low concentrating symmetric CPC:

$$H = r / 0.0645(2\theta_a)^{1.8494} \quad (7)$$

$$A_w = r / 0.0766(2\theta_a)^{0.964} \quad (8)$$

Where  $H$ ,  $r$ ,  $A_w$  and  $2\theta_a$  are the collector full height, receiver radius, aperture width and acceptance angle (in radian) respectively. The accuracy of equation 7 was tested by comparing the collector heights obtained from the program and those from the developed equation which gives a minimum and maximum deviation of -1.5

and 1.4mm respectively (i.e. less 1%). While that of equation 8 for the aperture width gives a minimum and maximum deviation of -0.63 and 0.48mm respectively. The correlation coefficient of equations 7 and 8 are 0.997 and 0.999 respectively.

Since the dimensions (height and aperture width) of the CPC are usually obtained after drawing the geometry, then these correlations can be important in giving idea on such dimensions of the symmetric CPC for a particular acceptance angle and receiver radius without necessarily drawing the profile. It can also be seen from Table 1 that height-to-aperture ratio and the geometric concentration ratio increase with the decrease in the acceptance angle but they are independent of the receiver radius. This shows that the increase in height due to the increase in the receiver size is proportional to the increase in the aperture width, thus by combining equations 7 and 8, the height can be expressed as:

$$H = 1.19A_w / (2\theta_a)^{0.8854} \quad (9)$$

It is shown from the foregoing results that the acceptance angle of the HPCPC is very important in defining the geometric characteristics and concentration ability of a CPC solar collector.

#### 4.0 CONCLUSION

Detailed analysis of the characteristics of HPCPC (with acceptance angles between 20° and 60°) was carried out by developing a computer program in Microsoft Excel spreadsheet. The program used parametric equations to determine the HPCPC geometric parameters like height, aperture width, height-to-aperture ratio, truncation effects, concentration ratio and average number of reflections. It also generates the HPCPC



profile for various inputs of acceptance angles and receiver radii. Results have shown that acceptance angle of the HPCPC is the most important parameter in determining its geometric characteristics because it affects the collector height and aperture, concentration ratio, average number of reflections, etc. New correlations relating collector height, aperture width and acceptance angles were proposed using the data obtained from such model.

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