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Modeling of the Optimum Tilt of a Solar Collector to Receive Maximum Radiation

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Abstract *- This work examines the theoretical aspects of choosing a tilt angle for the solar flat-plate collectors used at different stations, and makes recommendations on how the collected energy can be increased by varying the tilt angle. In this paper, the collector surface is assumed to be facing toward equator. For Kano stations, the calculations are based upon the measured values of monthly mean daily global and diffuse solar radiation on a horizontal surface. Also the calculations are based upon the data of monthly mean daily global solar radiation and monthly average clearness index on a horizontal surface. It is shown that nearly optimal energy can be collected if the angle of tilt is varied seasonally; four times a year. Annual optimum tilt angle is found to be approximately equal to latitude of the location. It is found that the loss in the amount of collected energy is around 1.5% if the angle of tilt is adjusted seasonally instead of using* β_{opt} *for each month of the year. The loss of energy when using the yearly average fixed angle is around 16 % compared with the monthly optimum tilt.*

Keywords - modeling, tilt angle, solar radiation, solar energy and solar collector

1. INTRODUCTION

Solar energy is a very large, inexhaustible source of energy. Quantitative assessment of solar radiation incident on a tilt plane is very important to engineers designing solar energy collecting devices, to architects designing buildings, and to agronomists studying insolation on vegetation on mountain slope. To address all these requirements, one should know the intensity of radiation falling upon sloping surface and its variation over a period of one year.

The most common form of solar energy utilization in the world is for water heating applications, mainly during the winter months. Generally conventional liquid flat-plate collectors are used in domestic hot water applications and installed on the roofs of many houses lying flat. These systems are manufactured and installed by ordinary craftsmen with little or no knowledge of solar or heat theory. These collectors are mounted with their surfaces facing towards the equator and the tilt angle is set approximately equal to the latitude of the location of interest.

The optimum tilt (inclination) of solar collector with respect to user is an important subject from application of thermal/electrical energy point of view. By utilizing maximum solar energy through the optimum tilt, it is possible we are able to harness the energy needed without polluting our environment, this reduces CO_2emissions in the atmosphere. Optimum tilt can be achieved by use of tracking systems. There are two types of tracking systems; Manual and Automatic.

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The following methods have been adopted in order to achieve optimum tilt:

- Monthly based optimization. Both manual and automatic tracking system can be used.
- Season based optimization: In this case also manual and automatic both tracking systems can be used. Automatic tracking system is expensive as compared to manual one.
- Annual based. In this case no need of automatic tracking system and hence, only manual tracking system is used.

Solar radiation data is usually measured in the form of global and diffuse radiation on a horizontal surface at the latitude of interest. Flat-plate solar collector are tilted so that they capture the maximum radiation and the problem of calculating solar radiation on a tilted surface is in determining the relative amount of beam and diffuse radiation contained in the measured horizontal global radiation. Since the flat plate solar- collectors are positioned at an angle to the horizontal, it is necessary to calculate the optimum tilt angle which maximizes the amount of collected energy. The best way to collect the maximum solar energy is by using solar tracking systems to follow the sun as it moves each day, and thus to maximize the collected beam radiation. It is possible to collect 40% more solar energy by using a two-axis tracking system (Markvatt,1994) and it is estimated that in sunny climates, a flat-plate collector moved to face the sun twice a day can intercept nearly 95% of the energy collected using a fully automatic solar tracking system (Markvatt,1994). Tracking systems are expensive, and need energy (usually solar energy is used) for their operation and they cannot easily be made applicable to solar collectors used for water heating purposes.

It is generally known that in the northern hemisphere, the optimum collector orientation is south facing and the optimum tilt depends upon the latitude and the day of the year. In winter months, the optimum till is greater, whilst in summer months; the optimum tilt is less. There are many papers in the literature which make different recommendations for the optimum tilt, based only on the latitude (Kern, 1997). In practice, the collector plate is usually oriented south facing and at a fixed tilt which is set to maximize the average energy collected over a year,

Chiou et al. (1986) gave a method to calculate the optimum tilt angle of an equator-facing collector in all seasons. (Kern, 1997) calculated the optimum tilt angle for equator-facing collectors, based on only beam radiation. El-Sayed (1989) has carried out an analysis to determine the optimum tilt angle by considering the effects of the latitude, number of glass covers, incident radiation and the solar reflectivity.

The theoretical aspects of choosing a tilt angle for the solar flat-plate collectors at different locations can be achieved by varying the tilt angle. A computer program was developed to simulate the collected energy as the tilt angle is varied.

1.1. Estimation Techniques

As most published meteorological data give the total radiation on horizontal surfaces, correlation procedures are required to obtain insolation values on tilted surfaces from horizontal radiation. Monthly average daily total radiation on a tilted surface (H_T) is normally estimated by individually considering the direct beam (H_B), diffuse (Hs) and reflected components (H_R) of the radiation on a tilted surface. Thus for a surface tilted at a slope angle from the horizontal, the incident total radiation is given by

$H_T = H_B + H_S + H_R$ (Hay, 1989) (1)

Several models have been proposed by various investigators (Hay, 1989 and Kamali et al, 2006) to calculate global radiation on tilted surfaces from the available data on a horizontal surface. The only difference among the models appears in the assessment of sky-diffuse component. Based on the assumptions made, the estimation models can be classified into isotropic (Liu et al, 1986) and anisotropic (Hay, 1989) ones. The daily beam radiation received on an inclined surface H_B can be expressed as

$H_B = (H - H_d) + H_R$ (Hay, 1989) (2)

Where H and H_d are the monthly mean daily global and diffuse radiation on a horizontal surface. The daily ground reflected radiation can be written as

$H_R = H\rho(1 - \cos\beta)/2$ (Hay, 1989) (3)

Liu et al, (1986) have suggested that R_b can be estimated by assuming that it has the value which would be obtained if there were no atmosphere. For surfaces in the northern hemisphere, sloped towards the equator, the equation for R_b is given as (Liu et al, 1962).

$$R_b = \frac{\cos(\emptyset-\beta)\cos\delta\,\sin\omega_s + \omega_s\sin(\emptyset-\beta)\sin\delta}{\cos\emptyset\,\cos\delta\,\sin\omega_s + \omega_s\,\sin\emptyset\,\sin\delta} \quad (4)$$

Where:

ω_s = min {cos^{-1}(- tan ∅ tan δ), cos^{-1}(- tan∅- β) (tan δ)} (5)

ω_s is the sunset hour angel for the tilted surface for the mean day of the month, "min" means the smaller of the two terms in the bracket.

∅ is the latitude, δ is the declination angle and ω is the angle from the local solar noon. The declination angle is given as (Miguel et al., 2001).

$\delta = 23{,}45\,\sin[360(284 + n)/365]$. (6)

Where n is the nth day of the year (ranging from 1 to 365).

Assuming isotropic reflection, the daily ground reflected radiation can be written as

$H_R = H\rho(1 - \cos\beta)/2$ (7)

1.2 Diffuse Radiation Models

The methods to estimate the ratio of diffuse solar radiation on a tilted surface to that of a horizontal are classified as isotropic and anisotropic models. The isotropic models assume that the intensity of diffuse sky radiation is uniform over the sky dome. Hence, the diffuse radiation incident on a tilted surface depends on the fraction of the sky dome seen by it. The anisotropic model assumed the anisotropy of the diffuse sky radiation in the circumsolar region (sky near the solar disc) and isotropically distributed diffuse component from the rest of the sky dome.

The sky-diffuse radiation can be expressed as

$H_s = R_d H_d$ (8)

Where$R_d$is the ratio of the average daily diffuse radiation on a tilted surface to that on a horizontal surface. The diffuse radiation models chosen for study were as follows (Hamdy et al, 2006).

1.3 Isotropic Models

Badescu (2002). model

$$H = H_b R_d + H\rho\left(\frac{1-\cos\beta}{2}\right) + H_d\left(\frac{3+\cos 2\beta}{4}\right)^2 \quad (9)$$

Koronakis (1986) model

$$H = H_b R_d + H\rho\left(\frac{1-\cos\beta}{2}\right) + H_d\left(\frac{2+\cos 2\beta}{3}\right) \quad (10)$$

Liu and Jordan (1962) model

$$H = H_b R_d + H\rho\left(\frac{1-\cos\beta}{2}\right) + H_d\left(\frac{1+\cos 2\beta}{2}\right) \quad (11)$$

1.4 Anisotropic Models

HDRK (2006) model

$H = (H_b + H_d A)R_b + H\rho\left(\frac{1-\cos\beta}{2}\right) + H_d\,\Big[(1 -$

A$1+cos2\beta2$1+sin3β2 (12)

Reindl et al., (1990) model

$$H = (H_b + H_d A)R_b + H\rho\left(\frac{1-cos\beta}{2}\right) + H_d\left[(1 -\right.$$
A $1+cos2\beta 21+HbHsin3\beta 2+A$ Rb (13)

Hay and Davies (1981) model

$$H = (H_b + H_d)R_b + H\rho\left(\frac{1-cos\beta}{2}\right) + H_d\left[\left(\frac{1+cos2\beta}{2}\right)(1-A)A\,R_b\right] \quad (14)$$

Where A *is Anisotropy index*

1.5 Total Radiation on a Tilted Surface

Total radiation on a tilted surface, can thus be expressed as

$$H_T = (H - H_d)R_b + H\rho\left(\frac{1-cos\beta}{2}\right) + H_d R_d$$ (Miguel et al, 2001) (15)

As no information is available on ground albedo, ρ values are assumed to be 0.2 when mean monthly temperatures is greater than 0° (Beckman et al, 1991). According to Equation (15), we need the direct and diffuse components of global radiation for estimating global solar radiation on tilled surfaces for this study these components were not available separately, so we used (Miguel et al., 2001) model to estimate the daily direct and diffuse components from measured daily global irradiance. Computer programs were written at various stages of the work. Data of monthly mean daily global radiation and monthly average clearness on horizontal surfaces needed for the study was taken from (Beckman et al, 1991).

1.6 Experimental Data

Data have been obtained using thermoelectric pyranometer. The pyranometer used are supposed to be calibrated once a year with reference to the World Radiometric Reference (WRR). Critical information such as calibration history, instrument changes, data quality control process, and shading due to obstructions in the horizon, is simply not available for these stations (Kano), "therefore it is to be expected that some data sites have larger uncertainties, with possibly more incorrect or missing data than others. But this situation is also confronted by any investigator using this kind of radiation data directly to design solar energy systems.

2. METHODOLOGY

Using Equations (1) to (18), total solar radiation falling on tilted surface was computed for tilt angle 0, 10, 20, 30, 40, 50, 60, 70, 80 and 90 degrees for each month of the year and for each station. Using MSEXCEL graphics software package, graphs were plotted between the total insolation versus tilt angle for each month and each station. Second order polynomial equations were developed to fit the curves. These polynomial equations were differentiated with respect to till angle and then equated to zero to obtain the optimum tilt angle corresponding to maximum insolation. Thus optimum tilt angle was computed for each month.

Computer programs in MATLAB were developed using the above formulae to calculate the monthly average daily total radiation on a surface facing towards equator as the tilt angle is changed from 0 to 90°. The solar reflectivity (ρ) was assumed to be 0.2.

3. RESULTS AND DISCUSSION

Fig. (1) Shows the monthly average daily global solar radiation H and the monthly average extra-terrestrial daily radiation H_0, on a horizontal surface in Kano (Latitude 12°N and Longitude 8°.31'E). The average value of H in Rainy season is 14.82 MJ/m^{2}day and its average value in Dry season is 22.97 MJ/m^{2}day. The beam and diffuse components of the monthly average daily global radiation on a horizontal surface are shown in Fig. (2). In winter months, the beam and diffuse components are nearly equal, and thus both components make the same contribution to the global radiation. In summer months, the beam component is more than diffuse component and thus the main contribution comes from the beam component. In monsoon season, the diffuse component is more than beam component.

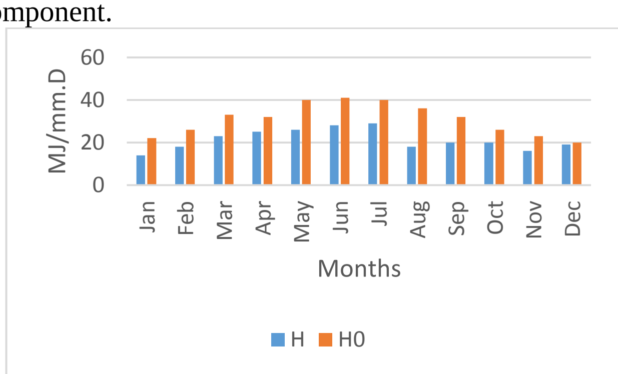

Fig. 1: Monthly average daily global radiation (H) and monthly averaged extra-terrestrial daily radiation (H_0) on a horizontal surface for Kano

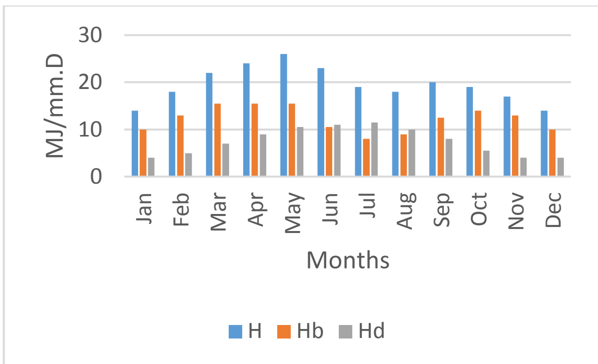

Fig. 2: monthly average daily global radiation (H), beam radiation (H_b) and diffuse radiation (H_d) on a horizontal surface for Kano

Fig. 1 and 2 shows the average daily total solar radiation on a south facing surface as the angle of till is varied from 0 to 90^0 in steps of 10^0. It is clear from these graphs that a unique β_{opt} exists for each month of the year for which the solar radiation is at a peak for the given month. Graphs for other stations have not been shown in order to avoid repetition. Table 1 gives a summary list of β_{opt} for each month of the year, the optimum angle of tilt of a flat-plate collector in January is 56° and the total monthly solar radiation falling on the collector surface at this tilt is 87.4MJ/m^2. The optimum till angle in June goes to a minimum of zero degree and the total monthly solar radiation at this angle is 796.3 MJ/m^2. The optimum tilt angle then increases during the winter months and reaches a maximum of 58° in December which collects 759.4 MJ/m^2 of solar energy monthly.

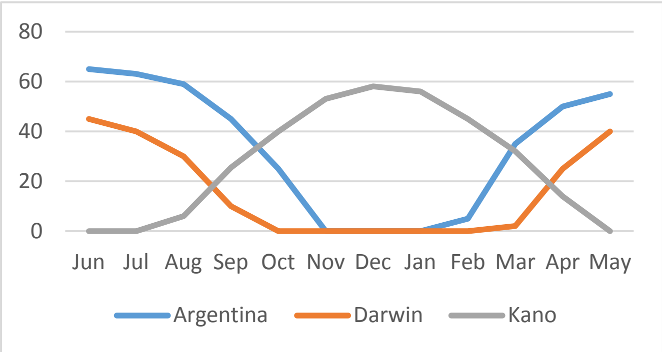

Fig. 3: Optimum average tilt angle for each month of the year for three different locations of northern hemisphere and tropical (Kano)

Fig. 3 shows the tilt angles for each month of the year when the collector panel is tilted at the optimum angle at different locations; Argentina, Darwin and Kano. The optimum average tilt angle for Kano reaches its minimum 0° for the months of June, July and May and the maximum of 58° in the month of December. When compared to the locations of Argentina and Darwin their angles are 0° respectively in the months of October, November December, January and February.

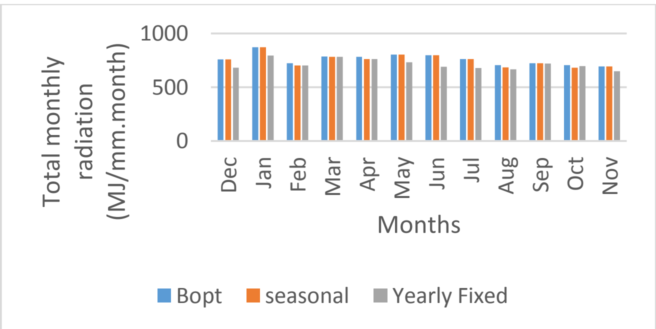

Fig.4: Total monthly solar radiation for Optimum seasonally adjusted and yearly fixed tilt angles for Kano station

Table 1: Optimum Tilt Angle (β_{opt}) for each Month of the Year for a South Facing Solar Collector for Kano

Month	$\boldsymbol{\beta_{opt}}$(Degrees)	Monthly Radiation (MJ/m^2, Month)
Jan	56	870.2940
Feb	45	721.6720
Mar	32	783.8598
Apr	14	780.8160
May	0	803.3030
Jun	0	795.8400
Jul	0	760.8950
Aug	6	705.4670
Sep	25.5	721.9230
Oct	40	704.8780
Nov	53	694.2570
Dec	58	758.8614

Table 2 shows the seasonal average, and the yearly average tilt angles. The seasonal average was calculated by finding the average value of the tilt angle for each season and the implementation of this requires the collector tilt to be changed four times a year. In spring the tilt should be $24^{0,}$ in summers zero degree, in autumn 30° and in winter 56°. The yearly average tilt was calculated by finding the average value of the tilt angles for all months of year. The yearly average tilt was found to be 30° and this result in a fixed tilt

Fig.4 shows the monthly solar energy collected when the angle of tilt is optimum, when the seasonal average angles of tilt are used, and when the yearly average angle is used throughout the year. The collected energy is tabulated in detail in table 2. When the monthly optimum tilt angle was used, the yearly collected solar energy was 9103 MJ/m^2. With the seasonally adjusted tilt angles, the yearly collected solar energy was 9015 MJ/m^2. Finally, with the yearly average tilt angle, the yearly collected solar energy was 7879 MJ/m^2.

Table 2: Optimum Tilt, seasonally Adjusted Tilt and Yearly Average Tilt and Monthly Solar Radiation on a Tilted South Facing plane, The Numbers inside Brackets are the Tilt Angles

S/N	Tilt	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Yearly
1	β_{opt}	870.29 (56)	721.67 (45)	783.86 (32)	780.82 (14)	803.30 (0)	795.84 (0)	760.89 (0)	705.47 (6)	721.92 (25.5)	704.88 (40)	694.26 (53)	758.86 58)	9102.1
2	Seasonally adjusted β	870.29 (56)	701.43 (30)	783.52 (30)	760.08 (30)	803.30 (0)	795.84 (0)	760.89 (0)	684.04 (24)	721.75 (24)	681.91 (24)	693.58 (56)	758.38 (56)	9015.0
3	Yearly average β	795.34 (30)	701.43 (30)	783.52 (30)	760.08 (30)	732.22 (30)	691.05 (30)	678.16 (30)	667.52 (30)	720.33 (30)	695.73 (30)	649.20 (30)	682.22 (30)	7878.6

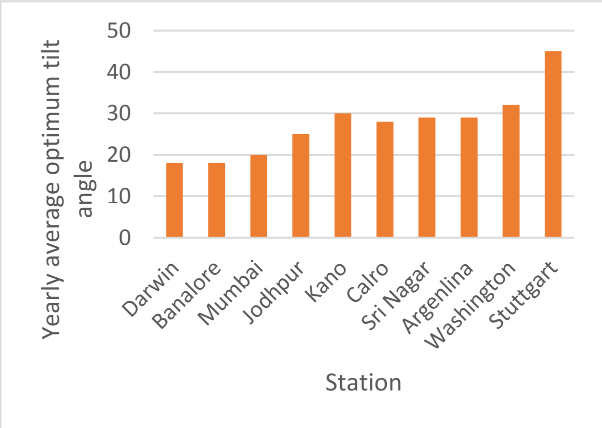

Fig. (5): Yearly average optimum tilt angle for different stations

Fig.5. Shows the yearly optimum tilt for different stations. Stuttgart has the highest reading of 45°, Kano with 30° and Darwin with the lowest of 18°.

4. CONCLUSION

Monthly based optimum tilt is different for different stations (fig. 3). Season based optimum tilt is also different for different stations (Fig. 3, Table 2). Annual based optimum tilt is approximately equal to latitude of the location. All the eight diffuse radiation models yield the same optimum tilt.

The results show that the average optimum tilt angle for the harmattan season months is 47.5^0 (latitude + 19^0) and for the hot season months 13^0 (latitude - 16^0). This, in general, is in agreement with the results of many other researchers (15, 16).

It is clear from table 2 that the loss in the amount of collected energy is less than 1% (0.97) if the angle of tilt is adjusted seasonally instead of using β_{opt} for each month of the year. The loss of energy when using the yearly average fixed angle is around 13.4% compared with the optimum tilt at New Delhi. It can be concluded that a yearly average fixed tilt can be used in many general applications (e.g. domestic water heating) in order to keep the manufacturing and installation costs of collectors low. For higher efficiency, the collector should be designed such that the angle of tilt can easily be changed at least on a seasonal basis, if not monthly. Alternatively, solar tracking systems can be used in industrial installations where higher efficiency is required.

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